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MULTIPLICITY RESULTS ON A FOURTH ORDER NONLINEAR ELLIPTIC EQUATION

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ABSTRACT. We are concerned with a fourth order semilinear elliptic boundary value problem under Dirichlet boundary condition $\Delta^2 u + c\Delta u = bu^+ + s$ in Ω, where Ω is a bounded open set in $\mathbf{R}^n$ with smooth boundary. We investigate the existence of solutions of the fourth order nonlinear equation when the nonlinearity bu^+ crosses eigenvalues of $\Delta^2 + c\Delta$ under the Dirichlet boundary condition and s is constant.

0. Introduction. Let Ω be a bounded open set in $\mathbf{R}^n$ with smooth boundary $\partial\Omega$. In this paper, we are concerned with a fourth order semilinear elliptic boundary value problem

$$\begin{aligned} \Delta^2 u + c\Delta u &= bu^+ + s \quad \text{in} \quad \Omega, \\ u = 0, \quad \Delta u &= 0 \quad \text{on} \quad \partial\Omega, \end{aligned} \tag{0.1}$$

where $u^+ = \max\{u, 0\}$, s is real, and c is not an eigenvalue of $-\Delta$ under Dirichlet boundary condition. The operator Δ^2 denotes the biharmonic operator. We assume that b is not an eigenvalue of $\Delta^2 + c\Delta$ under Dirichlet boundary condition.

The nonlinear equation with jumping nonlinearity has been extensively studied by many authors [**3, 4, 6, 7, 8**]. They studied the existence of solutions of the nonlinear equation with jumping nonlinearity for the second order elliptic operator [**6**], for one dimensional wave operators [**3, 4**], and for other operators [**7, 8**] when the source term is a multiple of the positive eigenfunction.

In [**13**], Tarantello considered the fourth order, nonlinear elliptic problem under the Dirichlet boundary condition

$$\begin{aligned} \Delta^2 u + c\Delta u &= b[(u+1)^+ - 1] \quad \text{in} \quad \Omega, \\ u = 0, \quad \Delta u &= 0 \quad \text{on} \quad \partial\Omega. \end{aligned} \tag{0.2}$$

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