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HYPERNORMAL FORMS FOR EQUILIBRIA OF VECTOR FIELDS. CODIMENSION ONE LINEAR DEGENERACIES

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ABSTRACT. The Poincare-Dulac-Birkhoff normal form theorem determines how much a vector field can be simplified, depending uniquely on its linear part. Nevertheless, taking into account the nonlinear terms, it is possible to obtain further simplifications in the classical normal form. In this paper we define the hypernormal forms, which are the simplest that we can achieve using $\mathcal{C}^\infty$-conjugation.

In practice, the computation of a hypernormal form requires the solution of some nonlinear equations. For this reason, we define the pseudohypernormal form, which is not as general as the hypernormal form, but its computation involves only linear equations.

We characterize the hypernormal forms using the theory of transformations based on the Lie transforms. As examples, we work out the two cases of codimension one linear degeneracies: saddle-node and Hopf singularities, using the method previously presented. Finally, in both examples, we consider additional simplifications that can be obtained using $\mathcal{C}^\infty$-equivalence.

1. Introduction. The normal form theory is a powerful tool for the analysis of local bifurcation problems near a nonhyperbolic equilibrium point. The underlying idea in this theory is to use near-identity transformations to remove, in the analytic expression of the vector field, the terms that are inessential in the local dynamical behavior.

The normal form theorem determines how it is possible to simplify the analytic expression of a vector field, taking uniquely into account the linear part of the vector field. Our goal is to show how to obtain further simplifications on the classical normal form, considering the nonlinear terms of the vector field.

We start with a brief summary of the basic ideas of the normal form theory. Consider the system

$$\dot{x} = f(x), \quad x \in \mathbf{K}^n, \tag{1.1}$$

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