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CYCLIC COMPOSITION OPERATORS ON SMOOTH WEIGHTED HARDY SPACES

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ABSTRACT. We determine hypercyclic and cyclic composition operators induced by a linear fractional self map of the unit disc, acting on a special class of weighted Hardy spaces. We establish the extreme possible cases and provide examples of spaces where they occur.

1. Introduction. Let H be a Hilbert space of functions analytic in the unit disc $\mathbf{D}$, and let ϕ be a nonconstant self map of $\mathbf{D}$. The *composition operator* C_ϕ on H is defined by $C_\phi f = f \circ \phi$ for all f in H.

When H is the classical Hardy space H^2, the operator C_ϕ is bounded. Some general properties of C_ϕ on H^2 are known, but still there are a lot of open basic questions. The situation becomes more complicated as we turn to some general classes of Hilbert spaces, for example, weighted Hardy spaces. Then it is still an open question precisely which analytic self maps of $\mathbf{D}$ will induce bounded composition operators. Nevertheless, composition operators provide a very interesting and important class of concrete examples of operators and, like multiplication operators, give a natural connection between operator theory and analytic function theory.

For an extensive reference on composition operators in general, see [**4**] and [**10**].

This paper deals with the problem of cyclicity of composition operators. Recall that the operator T on a Hilbert space H is *cyclic* if there is a vector f, called *cyclic vector*, whose orbit $\{T^N f | n \geq 0\}$ has a dense linear span in H, and T is *hypercyclic* if there is a vector, called *hypercyclic vector*, whose orbit is dense in H. Hypercyclicity is a much stronger property than cyclicity and, clearly, every hypercyclic operator is cyclic.

Bourdon and Shapiro have done an extensive study of cyclic and hypercyclic linear fractional composition operators on H^2, see [**1**] and [**2**].

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