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SPHERICAL ISOMETRIES ARE HYPOREFLEXIVE

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ABSTRACT. The result from the title is shown.

Let $L(H)$ denote the algebra of all bounded linear operators on a complex Hilbert space, H. If $\mathcal{M} \subset L(H)$, then we denote by $\mathcal{M}'$ the commutant of $\mathcal{M}$, $\mathcal{M}' = \{S \in L(H) : TS = ST \text{ for every } T \in \mathcal{M}\}$. The second commutant is denoted by $\mathcal{M}'' = (M')'$. Denote further by $\mathcal{W}(\mathcal{M})$ the smallest weakly closed subalgebra of $L(H)$ containing $\mathcal{M}$ and by $\operatorname{Alg}\operatorname{Lat}\mathcal{M}$ the algebra of all operators leaving invariant all subspaces which are invariant for all operators from $\mathcal{M}$. Recall that $\mathcal{M}$ is said to be *reflexive* if $\mathcal{W}(\mathcal{M}) = \operatorname{Alg}\operatorname{Lat}\mathcal{M}$. For a commutative set $\mathcal{M}$, there is also a weaker version of the reflexivity: $\mathcal{M}$ is called *hyporeflexive* if $\mathcal{W}(\mathcal{M}) = \operatorname{Alg}\operatorname{Lat}\mathcal{M} \cap \mathcal{M}'$.

Reflexivity and hyporeflexivity have been studied intensely by many authors. Deddens in [**3**] proved the reflexivity of a single isometry. The result was extended to sets of commuting isometries in [**2**], see also [**6**].

An analogy and, in some sense, a generalization of commuting N-tuples of isometries are spherical isometries. A *spherical isometry* is an N-tuple $T = (T_1, \dots, T_N)$ of mutually commuting operators on H satisfying $T_1^*T_1 + \dots + T_N^*T_N = I_H$.

The reflexivity of doubly commuting spherical isometries was mentioned in [**7**]. The aim of the paper is to show the hyporeflexivity of spherical isometries.

If μ is a positive Borel measure on the unit sphere

$$\partial\mathbf{B}_N = \{(z_1, \dots, z_N) \in \mathbf{C}^N : |z_1|^2 + \dots + |z_N|^2 = 1\},$$

then denote by $H^2(\mu)$ the closure of polynomials in $L^2(\mu)$. We start with the following

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