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ON THE ESSENTIAL SPECTRA OF REGULARLY SOLVABLE OPERATORS IN THE DIRECT SUM SPACES

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ABSTRACT. The problem of investigation of the spectral properties of the operators which are regularly solvable with respect to minimal operators $T_0(M_p)$ and $T_0(M_p^+)$ generated by a general quasi-differential expression M_p and its formal adjoint M_p^+ on any finite number of intervals $I_p = (a_p, b_p)$, $p = 1, \dots, N$, are studied in the setting of the direct sums of $L^2_{wp}(a_p, b_p)$-spaces of functions defined on each of the separate intervals. These results extend those of formally symmetric expression M studied in [**1**] and [**15**] in the single-interval case, and also extend those proved in [**10**] and [**13**] in the general case.

1. Introduction. Akhiezer and Glazman [**1**] and Naimark [**15**] showed that the self-adjoint extensions of the minimal operator $T_0(M)$ generated by a formally symmetric differential expression M with maximal deficiency indices have resolvents which are Hilbert-Schmidt integral operators and consequently have a wholly discrete spectrum. In [**10**], Ibrahim extended their results for general ordinary quasi-differential expression M of nth order with complex coefficients in the single-interval case with one singular endpoint.

The minimal operators $T_0(M)$ and $T_0(M^+)$ generated by a general ordinary quasi-differential expression M and its formal adjoint M^+, respectively, form an adjoint pair of closed, densely-defined operators in the underlying L^2_w-space, that is, $T_0(M) \subset [T_0(M^+)]^*$. The operators which fulfill the role that the self-adjoint and maximal symmetric operators play in the case of a formally symmetric expression M are those which are regularly solvable with respect to $T_0(M)$ and $T_0(M^+)$. Such an operator S satisfies $T_0(M) \subset S \subset [T_0(M^+)]^*$ and, for some $\lambda \in \mathbf{C}$, $(S-\lambda I)$ is a Fredholm operator of zero index, this means that S has the desirable Fredholm property that the equation $(S - \lambda I)u = f$ has a solution if and only if f is orthogonal to the solution space of

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