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DUALITY IN NOETHERIAN INTEGRAL DOMAINS

H. PAT GOETERS

A premier paper on torsion-free abelian groups was published by Warfield [**25**], the duality aspect of which can be summarized as follows. Given a torsion-free abelian group A of rank one, put $\mathbf{C}_A$ equal to the class of torsion-free abelian groups M of finite rank such that M embeds as an $\operatorname{End}(A)$ submodule of A^n for some n. Warfield shows that a torsion-free abelian group M of finite rank satisfies $M \cong_{\text{nat}} \operatorname{Hom}(\operatorname{Hom}(M, A), A)$ exactly when M belongs to $\mathbf{C}_A$. In functorial terminology, he shows that for any torsion-free rank one group A, the map $M \mapsto \operatorname{Hom}(M, A)$ on $\mathbf{C}_A$ defines a duality.

Reid was interested in extending Warfield's result to more general domains in an effort to classify his irreducible groups. Reid gave sufficient conditions in [**22**] for an arbitrary integral domain to support Warfield duality, conditions that will receive further attention below. Given a general torsion-free abelian group of finite rank, in order to understand when $\operatorname{Hom}(-A) : \mathbf{C}_A \to \mathbf{C}_A$ defines a rank preserving duality, one must know when $\operatorname{End}(A)$ supports Warfield duality [**11, 12**]. This enhances the desire to investigate extensions of Warfield duality to Noetherian domains.

For an integral domain R and a torsion-free module A of rank one, as above, let $\mathbf{C}_A$ represent the category of modules M isomorphic to $\operatorname{End}_R(A)$-submodules A^n for some n. Call a module M, *A-reflexive*, if $M \cong_{\text{nat}} \operatorname{Hom}(\operatorname{Hom}(M, A), A)$ (the unadorned $\operatorname{Hom}(M, A)$ will be used when the ring R is prescribed). In [**6**], an integral domain R is called a *Warfield* domain if, for any rank one module A, $\operatorname{Hom}(-, A) :$ $\mathbf{C}_A \to \mathbf{C}_A$ defines a duality. It is shown in [**9**] that, when R is a Noetherian domain whose integral closure is finitely generated over R, then R is Warfield if and only if every ideal of R is two-generated. The restriction on the integral closure in [**9**] was needed to show that, when R is a local domain such that every ideal is two generated, and A is a rank one R-module with endomorphism ring R, then A is finitely generated over R.

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