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# SOME ASPECTS OF DENTABILITY IN BITOPOLOGICAL AND LOCALLY CONVEX SPACES

MISHEL FUNDO

ABSTRACT. This paper is a continuation of the study we presented in [**6**]. A modified version of Namioka's argument is reconsidered to obtain an extended form of a result of Namika-Asplund; this leads to the improvement of several theorems and to a generalized version of the Dunford Pettis theorem [**2**]. Moreover, two versions of Rieffel's converse theorem are discussed. It is shown that the first one holds true in locally convex spaces, but not generally in the spaces with two topologies; the second leads to a new characterization of the Radon-Nikodym property in real Banach spaces and in their duals.

**1. Introduction.** The main results are contained in Sections 2 and 3. Section 2 deals with sufficient conditions of $\tau$-dentability in bitopological spaces [**13**]. Corollaries in locally convex spaces are obtained. Analogous questions have been treated in a different context by Rieffel [**15**], Maynard [**12**], Lindenstrauss [**11**], Troyanski [**17**], Bourgain [**1**], Namioka [**13**], etc.

Section 3 deals with the problem of equivalence between the dentability of a bounded set and the dentabilities of its closed convex hull and its closed equilibrated convex hull; the aim is to find the possibly larger extension of such an equivalence. This is an important tool in the study of the geometric aspects of the Radon-Nikodym property.

In this paper the same notations and definition as in [**6**] are used. Thus we recall some of them by paying special attention to the notion of bitopological spaces introduced by Namioka [**13**].

We consider a real vector space $E$ with two topologies $r_0$ and $r$ such that $r < r_0$, and we suppose that the pairs $(E, r_0) = E$ and $(E, r) = E_r$ are Hausdorff locally convex spaces (hlcs); their topological duals are denoted by $E'$ and $E'_r$, respectively, and the system of neighborhoods of the origin in $E$ by $\gamma(0)$. The weak-compact sets in $E_r$ will be called

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