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DIRECT SUMS AND SUMMANDS OF WEAK CS-MODULES AND CONTINUOUS MODULES

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Introduction. In [**5**] it is left as a question whether direct sums and summands of weak CS-modules are weak CS or not. Some particular answers are given to the former question in [**5**, Lemma 1.10, Lemma 1.11, Theorem 1.12], and in the first part of this note we give a general result, Theorem 1.9, of which those assertions are corollaries, as well as the assertion that a finite direct sum of relatively injective weak CS-modules is weak CS, Corollary 1.10, the dual of which is proved for CS-modules by Harmanci and Smith in [**2**]. As for the latter one we give an affirmative answer for a module with C_3 property and a UC [**6**], in particular, nonsingular, module. Finally, in this section, we give a sufficient condition for a nonsingular module to be CESS. In the second part some properties of weak CS-modules in common with modules satisfying C_{11} [**8**] are investigated and a class of modules, direct summands of which are direct sums of uniform modules, Proposition 2.6, is introduced. In the third part a generalization of continuous modules is given, namely F-modules. Continuous modules are characterized in terms of F-modules satisfying the C_{11}-property. We eventually prove that a direct sum M of C_{11}, hence CS/continuous, modules is continuous if and only if M is an F-module.

In this paper R will denote a ring with identity and M a unitary right R-module. For any submodule K of M, the family of submodules N satisfying $K \cap N = 0$ has a maximal member by Zorn's lemma, which is called a *complement of* K *in* M. A submodule N of M is called a *complement in* M if N is a complement of a submodule of M. It is easy to see that a submodule is a complement in M if and only if it has no proper essential extensions in M. For $m \in M$, the *right annihilator of* m is the set of elements r of R such that $mr = 0$, and is denoted $ann(m)$. A module M is called *nonsingular* if no element of M except 0 has annihilator which is essential in R_R. A module M is said to be a CS-*module* if every complement in M is a direct summand of M,

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