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ALMOST SURE CONVERGENCE AND DECOMPOSITION OF MULTIVALUED RANDOM PROCESSES

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ABSTRACT. This paper examines multi-valued random processes (random sets) with values in a separable Banach space. Several results on the almost sure convergence and decomposition properties of various classes of random processes are established. Special consideration is given to multi-valued submartingales, uniform amarts, weak sequential amarts and amarts of infinite order. In the process some results concerning sequences of vector-valued random variables are also proved.

1. Introduction. Multi-valued random processes and more specifically multi-valued discrete time martingales, were first introduced in the early seventies by Van Cutsem [**35, 36**] in connection with problems of stochastic optimization. Since then the subject has attracted the interest of many mathematicians and further contributions were made from both the theoretical and applied viewpoints. The development of the theory can be traced in the works Neveu [**26**], Daures [**11**], Hiai and Umegaki [**18**], Coste [**10**], Luu [**23, 24**], Bagchi [**3, 4, 5**], Papageorgiou [**28, 30, 31**], de Korvin and Kleyle [**20**], Hess [**17**] and Wang and Xue [**38**]. Applications to stochastic optimization, mathematical economics and information systems can be found in the works of Arstein and Hart [**1**], Salinetti and Wetts [**33**], Papageorgiou [**27**] and Yovits et al. [**40**].

In this paper we make some further contributions to the convergence and decomposition theories of set-valued random processes (random sets). We consider a variety of multi-valued random processes starting with submartingales. Via a "weak* compactness" result for multifunctions, we prove a submartingale convergence theorem which extends the result of Neveu [**26**].

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