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FINITE CODIMENSIONAL INVARIANT SUBSPACES OF BANACH SPACES OF ANALYTIC FUNCTIONS

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ABSTRACT. Let G be a bounded domain in the complex plane. Let $\mathcal{E}$ be a Banach space of functions analytic on G, such that for each $\lambda \in G$ the linear functional e_λ of evaluation at λ is bounded on $\mathcal{E}$. Assume further that $z\mathcal{E} \subset \mathcal{E}$ and, for every $\lambda \in G$, $\operatorname{ran}(M_z - \lambda) = \ker e_\lambda$. Here M_z is the operator of multiplication by z on $\mathcal{E}$ given by $f \mapsto zf$. In this article we characterize the finite codimensional subspaces of $\mathcal{E}$ which are invariant under M_z in some special cases.

1. Introduction. Let G be a bounded domain in the complex plane. Let $\mathcal{E}$ be a Banach space of functions analytic on G such that for each $\lambda \in G$ the linear functional e_λ of evaluation at λ is bounded on $\mathcal{E}$. Assume further that $z\mathcal{E} \subset \mathcal{E}$ and for every λ in G, $\operatorname{ran}(M_z - \lambda) = \ker e_\lambda$. A Banach space $\mathcal{E}$ with all the above properties is called a Banach space of analytic functions and is called a Banach space of functions if we only have $z\mathcal{E} \subset \mathcal{E}$. As a result we conclude that $M_z - \lambda$ is Fredholm for every $\lambda \in G$ and because $\dim \ker(M_z^* - \lambda) = 1$ we have $\operatorname{ind}(M_z - \lambda) = -1$ for $\lambda \in G$. A function $\varphi : G \to \mathbf{C}$ with the property $\varphi\mathcal{E} \subset \mathcal{E}$ is called a *multiplier* on $\mathcal{E}$, and the collection of all these multipliers is denoted by $\mathcal{M}(\mathcal{E})$. If $\varphi \in \mathcal{M}(\mathcal{E})$, then the operator M_φ of multiplication by φ is bounded.

Richter [**11**] has shown that the commutant of the operator M_z is equal to $\{M_\varphi : \varphi \in \mathcal{M}(\mathcal{E})\}$. This makes $\mathcal{M}(\mathcal{E})$ into a Banach space by defining $\|\varphi\|_{\mathcal{M}(\mathcal{E})} = \|M_\varphi\|_{\mathcal{L}(\mathcal{E})}$. It is also true that $\mathcal{M}(\mathcal{E}) \subset H^\infty(G)$ and for each $\varphi \in \mathcal{M}(\mathcal{E}), \|\varphi\|_\infty \le \|M_\varphi\|_{\mathcal{L}(\mathcal{E})} = \|\varphi\|_{\mathcal{M}(\mathcal{E})}$. Now suppose that $\mathcal{M}(\mathcal{E})$ contains a norm closed subalgebra $\mathcal{A}$ of $H^\infty(G)$. Then the above inequality shows that $\mathcal{A}$ is also closed in $\mathcal{M}(\mathcal{E})$ and the open

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