ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 29, Number 3, Fall 1999

ON THE NONEXISTENCE OF COFREE FRÉCHET MODULES OVER LOCALLY MULTIPLICATIVELY-CONVEX FRÉCHET ALGEBRAS

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ABSTRACT. Suppose A is a locally multiplicatively-convex Fréchet algebra. It is proved that, if there exists at least one nonzero, cofree Fréchet A-module, then A is normable.

Topological homology is based on two fundamental concepts: projectivity and injectivity. These concepts can be defined in the context of locally convex modules over a locally convex algebra A, see [**3**]. If A is a Banach algebra, then many important statements about projective and injective modules, known from classical homological algebra, are valid in the categories of Banach A-modules. In particular, each Banach A-module has projective and injective resolutions, see [**3**]. However, if A is an arbitrary locally convex algebra, then only the notion of a projective A-module is rich in content. The main obstacle for the study of injective A-modules is the possible absence of so-called *cofree* objects in categories of locally convex A-modules. Even if A is a nonnormable Fréchet algebra, we do not have any information about injective Fréchet A-modules. In this connection, the following question is of interest. Do there exist nonzero, cofree Fréchet A-modules over an arbitrary Fréchet algebra A?

This problem is closely connected with some questions concerning injective Fréchet A-modules, for example, is it true that any A-module has an injective resolution? Is it true that each injective A-module is a retract of a cofree A-module? Finally, does there exist at least one nonzero, injective A-module? See [**4**]. If A is a Banach algebra and the A-modules which are under consideration are also Banach modules, then the answers to all these questions are affirmative, as well as in pure homological algebra. In particular, a cofree Banach A-module is topologically isomorphic to an A-module $\mathcal{B}(A, E)$ for some Banach space E (by $\mathcal{B}(A, E)$ we denote the A-module of all continuous linear

Received by the editors on March 29, 1997, and in revised form on January 22, 1998.