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CONSTANT MEAN CURVATURE SURFACES BOUNDED BY A CIRCLE

RAFAEL LÓPEZ

Introduction. The structure of the space of compact constant mean curvature surfaces with prescribed boundary is not known, even in the simplest case: when the boundary is a round circle with, for instance, unit radius. Heinz [**4**] found that a necessary condition for existence in this situation is that $|H| \leq 1$. The only known examples are the umbilical ones: the flat disc if $H = 0$ and the two spherical caps with radius $1/|H|$ if $H \neq 0$; and some non-embedded surfaces of genus bigger than two whose existence was proved by Kapouleas in [**7**].

We shall consider a connected compact surface Σ and $\phi : \Sigma \to \mathbf{R}^3$ an immersion of constant mean curvature H such that $\phi : \partial\Sigma \to \phi(\partial\Sigma)$ is a diffeomorphism. We will say in this situation that Σ is an *H-surface with boundary* Γ, where $\Gamma = \phi(\partial\Sigma)$.

When the boundary is a circle of radius one, we shall suppose that it is in the z-plane, and we shall denote by S^1 the circle $\{(x, y, 0) \in \mathbf{R}^3; x^2 + y^2 = 1\}$. Given $0 < |H| \leq 1$, the two spherical caps bounding S^1 are stable, but it is not known if they are the only ones bounded by S^1. There is no even answer to this question for immersed discs. In [**2**] it appears the question to find sufficient conditions of stability for a domain. Following ideas of Ruchert [**9**], Barbosa and do Carmo prove that if Σ is a simply-connected surface immersed in $\mathbf{R}^3$ with constant mean curvature and $\int_\Sigma |\sigma|^2 \, d\Sigma < 8\pi$, then Σ is stable, where σ denotes the second fundamental form. Also repeated with the problem of stability, Koiso [**8**] has proved that the spherical caps are the only surfaces with minimum area in the family of surfaces with constant volume and boundary S^1. In this paper we prove the following result.

Let $\phi : \Sigma \to \mathbf{R}^3$ be an immersion from a compact disc in $\mathbf{R}^3$ with constant mean curvature and such that $\phi(\partial\Sigma)$ is a circle of radius one. If σ is the second fundamental form and $\int_\Sigma |\sigma|^2 \, d\Sigma \leq 8\pi$, then ϕ is umbilical.

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