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A NOTE ON CYCLOTOMIC POLYNOMIALS

MING-CHANG KANG

ABSTRACT. Let R be a Dedekind domain and n a square-free positive integer, $\Lambda := R[T]/T^n - 1$. The fact that Λ is a Dedekind-like ring will make possible the classification of integral representations of the cyclic group of order n over R [**7**]. We shall prove that Λ is a Dedekind-like ring for a fairly large class of Dedekind domains R. The proof is facilitated by an identity among cyclotomic polynomials [**2**]. Some other applications of the same identity will be presented also.

1. Introduction. Let $\Phi_n(X)$ be the nth cyclotomic polynomial. Then the formula

$$X^n - 1 = \prod_{d|n} \Phi_d(X) \tag{1}$$

is well known. There are other identities among cyclotomic polynomials, which are not so well known, for example, identities of Beeger and Schinzel [**12, 1**]. Recently a new factorization identity was proved by Cheng, McKay and Wang [**2**], namely,

Theorem 1.1 [**2**, Lemma 1][**10**, p. 105] [**5**, p. 394]. *Let m, n, k be positive integers so that g.c.d. $\{m, n\} = 1$ and m is divisible by every prime factor of k. Then*

$$\Phi_m(X^{nk}) = \prod_{d|n} \Phi_{mkd}(X). \tag{2}$$

In particular, we find that

$$\Phi_m(X^n) = \prod_{d|n} \Phi_{md}(X), \text{if g.c.d. } \{m, n\} = 1; \tag{3}$$

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