ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 29, Number 3, Fall 1999

ON BOUNDARY CONDITIONS FOR STURM-LIOUVILLE DIFFERENTIAL OPERATORS IN THE DIRECT SUM SPACES

SOBHY EL-SAYED IBRAHIM

ABSTRACT. Sturm-Liouville (S-L) boundary value problems on any finite number of intervals are studied in the setting of the direct sum of the L^2_w-spaces of functions defined on each of the separate intervals. The interplay between these L^2_w-spaces is of critical importance. This study is partly motivated by the occurrence of (S-L) problems with coefficients that have a singularity in the interior of the basic interval. In the one interval case, the singular self-adjoint boundary conditions are characterized in terms of certain Wronskians involving y and two linearly independent solutions of $M[y] = 0$ by Krall and Zettl in [**11**].

1. Introduction. The boundary value problems for the Sturm-Liouville (S-L) expression

$$M[y] = \frac{1}{w}[-(py')' + qy] \quad \text{on } I = (a, b),$$
$$-\infty \le a < b \le \infty$$

on two intervals are studied in the setting of the direct sum of the L^2-spaces of functions defined on each of the separate intervals by Everitt and Zettl in [**8**]. In the one interval case, the characterization of singular self-adjoint boundary conditions for Sturm-Liouville problems is identical to that in the regular case provided that y and py' are replaced by certain Wronskians involving y and two linearly independent solutions of $M[y] = 0$ has been proved by Krall and Zettl in [**11**].

Our objective in this paper is to extend the results of Krall and Zettl in [**11**] to the case of any finite number of intervals $I_r = (a_r, b_r)$, $r = 1, 2, \dots, n$. Here the interior singularities occur only at the ends of the intervals. In particular, we define a minimal and a maximal operator each associated with expressions, and characterize all self-adjoint extensions of the minimal operator in terms of "boundary

Received by the editors on May 6, 1996.