

A NEW CLASS OF WEAKLY SYMMETRIC SPACES

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ABSTRACT. We prove that any simply connected Riemannian manifold which is equipped with a complete unit Killing vector field such that the reflections with respect to the flow lines of that field can be extended to global isometries, is a weakly symmetric space.

1. Introduction. Weakly symmetric spaces have been introduced by A. Selberg [23] in 1956. They may be characterized as connected Riemannian manifolds on which any two points can be interchanged by an isometry. Every Riemannian symmetric space is weakly symmetric, but the converse is not true. However, weakly symmetric spaces have many properties enjoyed by symmetric spaces. For example, in [23] it is proved that the algebra of all isometry-invariant differential operators on a weakly symmetric space is commutative and in [1], the authors show that all their geodesics are orbits of one-parameter groups of isometries of the manifold. Other geometric properties have been considered in [2, 5, 6].

At this moment many examples of nonsymmetric weakly symmetric spaces are known. We refer to [3, 4, 7, 14, 15, 17, 23, 29] for a detailed description. In particular, it has been proved in [7] that any simply connected φ -symmetric space is a weakly symmetric space. Such spaces have been introduced in [24] in the framework of contact geometry where they play a similar role as the Hermitian symmetric spaces in complex geometry. They provide examples of Riemannian manifolds which are equipped with a complete unit Killing vector field such that the reflections with respect to the flow lines can be extended to global isometries. Such Riemannian manifolds are called Killing-transversally symmetric spaces. Their local and global geometry as well

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