

BLOW-UP OF SOLUTIONS OF SOME NONLINEAR HYPERBOLIC SYSTEMS

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ABSTRACT. We consider two hyperbolic systems: $u_{tt} = \Delta u + |v|^p$, $v_{tt} = \Delta v + |u|^q$ and $u_{tt} = \Delta u + |v_t|^p$, $v_{tt} = \Delta v + |u_t|^q$ in $\mathbf{R}^n \times (0, \infty)$ with $u(x, 0) = f(x)$, $v(x, 0) = h(x)$, $u_t(x, 0) = g(x)$, $v_t(x, 0) = k(x)$. We show that there exists a bound $B(n, p)$ such that if $1 < pq < B(n, p)$ all nontrivial solutions with compact support blow up in finite time.

1. Introduction. In this paper we study two systems of hyperbolic equations:

$$(1.1) \quad \begin{aligned} u_{tt} &= \Delta u + |v|^p, & v_{tt} &= \Delta v + |u|^q, \\ u(x, 0) &= f(x), & v(x, 0) &= h(x), \\ u_t(x, 0) &= g(x), & v_t(x, 0) &= k(x), \\ x &\in \mathbf{R}^n, & t &> 0, \end{aligned}$$

and

$$(1.2) \quad \begin{aligned} u_{tt} &= \Delta u + |v_t|^p, & v_{tt} &= \Delta v + |u_t|^q, \\ u(x, 0) &= f(x), & v(x, 0) &= h(x), \\ u_t(x, 0) &= g(x), & v_t(x, 0) &= k(x), \\ x &\in \mathbf{R}^n, & t &> 0, \end{aligned}$$

where $p, q \geq 1$ and $pq > 1$, and the initial values are compactly supported. Such systems are special cases of a significant class of quasilinear second order hyperbolic systems with application in physics and applied science, cf. [5].

Our main objective here is to establish blow-up theorems for systems (1.1) and (1.2). As an example of the type of results we wish to obtain,

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