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MULTIVARIATE POLYNOMIAL SPLINE SPACES

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ABSTRACT. In this paper we construct bases of certain spaces of multivariate polynomial splines defined on rectangular partitions. These bases are presented by polynomials, truncated power functions and products of these functions. This setting provides a natural generalization of the two-dimensional polynomial spline, proposed by Chui and Wang [**3**], to p variables.

1. Introduction. A standard way of approximating the cause-and-effect relationship is using a single model over the entire range of variables, for example, the models for linear or polynomial functions. In practice, however, it might be more realistic to partition the range of variables into disjoint regions, and to approximate the relationship by a sequence of submodels which are smoothly connected, in some sense, at the boundaries of the neighboring regions. Polynomial spline functions are useful for this purpose.

Polynomial spline functions are generally defined as piecewise polynomials of degree k, whose partial derivatives satisfy certain smoothness conditions. Theoretical as well as applied research on univariate polynomial spline spaces has widely developed in the last two decades, see, for example, [**5**] and [**7**]. For bivariate polynomial splines, progress has also been made in the study of their bases and dimensions, see, for example, [**2, 3, 4, 6**] and a review article by [**8**]. For general multivariate polynomial splines, Alfeld, Schumaker and Sirvent [**1**] have studied the dimension and existence of local bases in triangulation partition. However, only the basis with zero smoothness-degree has been found. Therefore, development of the theory of multivariate polynomial splines, analogous to the well-known theory of univariate polynomial splines, has still remained to be achieved, see also [**5**, p. 362] and Schumaker [**8**, p. 195] for this point. Besides the need for its theoretical

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