ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 29, Number 4, Winter 1999

QUOTIENT MAPS WITH STRUCTURE PRESERVING INVERSES

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ABSTRACT. It is proved that certain quotient maps $q : (\sum_n U_n)_{l_1} \to Y$, where U_n are finite dimensional spaces, have the following property: If E is a subspace of Y with a "good" structure of uniformly complemented finite dimensional subspaces, so is the subspace $q^{-1}(E)$ of $(\sum_n U_n)_{l_1}$. In particular, any quotient map $q : l_1 \to L_1$ has this property.

1. Introduction. Let $q : U \to Y$ be a quotient map. In general, very little is known about the connection between a subspace E of Y and the subspace $q^{-1}(E)$ of U. In this note we discuss a quotient map q, the inverse of which preserves the π property and the finite dimensional decomposition property. Recall that a space E is said to be a π_λ space, $\lambda \geq 1$, if there exist a sequence $\{E_n\}_{n=1}^\infty$ of finite dimensional subspaces of E, with $E_1 \subset E_2 \subset \cdots$ and $\overline{\cup_{n=1}^\infty E_n} = E$, and a sequence of projections $\{P_n\}_{n=1}^\infty$ of E onto E_n with $\sup_n \|P_n\| = \lambda < \infty$. E is said to be a π space (or, to have the π property) if it is a π_λ space for some $\lambda \geq 1$. The pair of sequences $(\{E_n\}_{n=1}^\infty, \{P_n\}_{n=1}^\infty)$ will be called a π *structure of* E. If E has a π structure $(\{E_n\}_{n=1}^\infty, \{P_n\}_{n=1}^\infty)$ and, for every $n, k \geq 1$, $P_nP_k = P_kP_n = P_{\min(k,n)}$, then the sequence $\{(P_n - P_{n-1})(E)\}_{n=1}^\infty$ is called a finite dimensional decomposition of E, f.d.d. for short, and E is said to have the f.d.d. property.

Our main result is the following

Theorem. *Let Y be a π_λ space with a π_λ structure $(\{Y_n\}_{n=1}^\infty, \{Q_n\}_{n=1}^\infty)$, and let $U = (\sum_{n=1}^\infty Y_n)_{l_1}$. For each $n \geq 1$, let U_n denote the subspace $\{(\underbrace{0, \dots, 0}_{n-1}, y, 0, \dots) \in U : y \in Y_n\}$, and denote by τ_n the*

Received by the editors on September 1, 1997, and in revised form on May 15, 1998.
Participant at Workshop in Linear Analysis and Probability, NSF DMS-9311902.
AMS *Mathematics Subject Classification.* 46B25.
Partially supported by the Edmund Landau Center for Research in Mathematical Analysis, sponsored by the Minerva Foundation (Germany).