ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 29, Number 4, Winter 1999

FOR $b \geq 3$ THERE EXIST INFINITELY MANY BASE b k-SMITH NUMBERS

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ABSTRACT. In [**5**] a Smith number is defined as a composite, the sum of whose digits equals the sum of the digits of the prime factors counted with multiplicity. In [**1**] it was shown there are infinitely many Smith numbers by giving a constructive algorithm. In [**2**] this result was extended to Smith numbers in bases $b \geq 8$ and in [**3**] to $b = 2$. In this paper we modify the argument in [**2**] to cover all bases $b \geq 3$.

1. Introduction, notation and definitions. Let $b \geq 3$ be a fixed base. All our references to integers will be to integers base b unless otherwise noted. Let x be a natural number, and let $x = p_1 p_2 \cdots p_r$ be its factorization into not necessarily distinct primes. Let $N(x)$ denote the number of digits of x base b, $S(x)$ the sum of the base b digits of x and $S_p(x)$ the sum of the base b digits of the prime factors of x counted with multiplicity, i.e., $S_p(x) = \sum_{i=1}^{r} S(p_i)$. Note that

$$S_p(xy) = S_p(x) + S_p(y). \tag{1}$$

Let k be a natural number.

Definition. A *base b k-Smith number* is a composite x so that

$$S_p(x) = kS(x).$$

2. Preliminary results. We now cite a number of useful lemmas from [**2**].

Lemma 1. *If $m > 1$ there exists a t so that $S_p(t) = m$.*

In view of (1) this lemma says that, given natural numbers x and y so that $S_p(x) < y - 1$, we can find t so that $S_p(xt) = y$. In particular, we

Received by the editors on February 5, 1997, and in revised form on January 9, 1998.