ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 29, Number 4, Winter 1999

MODULES FOR WHICH HOMOGENEOUS MAPS ARE LINEAR

A.B. VAN DER MERWE

ABSTRACT. Given an R-module V, the near-ring of homogeneous maps $\mathcal{M}_R(V)$ is the set of maps $\{f : V \to V \mid f(rv) = rf(v) \text{ for all } r \in R \text{ and } v \in V\}$ endowed with point-wise addition and composition of functions as multiplication. Modules with the property that $\mathcal{M}_R(V) = \mathrm{End}_R(V)$ when R is commutative and Noetherian, and V is finitely generated, are characterized. Commutative Noetherian rings with the property that $\mathcal{M}_R(V) = \mathrm{End}_R(V)$ for all uniform modules, V, are also classified.

1. Introduction. Let R be a commutative Noetherian ring with identity and V a nonzero unital R-module. The set of maps $\mathcal{M}_R(V) := \{f : V \to V \mid f(rv) = rf(v) \text{ for all } r \in R \text{ and } v \in V\}$ is a right near-ring under point-wise addition and composition of functions, and the elements are called *homogeneous maps*. This near-ring has been the subject of several investigations. See, for example, [**3**] and [**4**]. We write functions on the left of the elements on which they act; therefore $\mathcal{M}_R(V)$ satisfies the right distributive law. Recall that an R-module V is *uniform* if for any nonzero R-submodules M and N, $M \cap N \neq \langle 0 \rangle$. In the third section, we will see in particular that modules over Dedekind domains are rather well behaved, since V uniform implies in this case that $\mathcal{M}_R(V) = \mathrm{End}_R(V)$. In fact, if we restrict ourselves to domains, this property will characterize Dedekind domains. From this consideration, we conclude that the problem of determining when homogeneous maps are linear becomes significantly more interesting when we consider Noetherian rings in general.

2. When are all the homogeneous maps on a finitely generated module linear? We denote the injective hull of V by $E(V)$. Since every module can be embedded in an injective module, the fol-

Received by the editors on November 4, 1994, and in revised form on June 22, 1995.
1991 AMS *Mathematics Subject Classification.* 16D70, 16S50, 16Y30.