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THE LATTICE OF SEMILATTICE-MATRIX DECOMPOSITIONS OF A SEMIGROUP

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ABSTRACT. In this paper we investigate some general, lattice theoretical properties of semilattice-matrix decompositions of semigroups. We prove that the poset of all semilattice-matrix equivalences on an arbitrary semigroup is a complete lattice. For a fixed semilattice congruence σ on a semigroup S we prove that the set of all semilattice-matrix equivalences on S carried by σ is a complete sublattice of the lattice of equivalence relations on S, and that it is a direct product of the lattices of semilattice-left and semilattice-right equivalences on S carried by σ.

Semilattice-matrix decompositions of semigroups, including here semilattice-left and semilattice-right decompositions, form a very important type of decompositions studied in many papers. We can say that the first known example of such decompositions was the characterization of unions of groups (completely regular semigroups) as semilattices of completely simple semigroups, given by A.H. Clifford in [**7**], 1941, since by the well-known Rees-Sushkevich matrix representation theorem, completely simple semigroups can be characterized as matrices (rectangular bands) of groups, left zero bands of right groups and right zero bands of left groups. A similar property was proved by Bogdanović and Ćirić in [**5**] for left regular semigroups, which were characterized as semigroups having a semilattice-right decomposition whose components are left simple semigroups. Semilattice-right decompositions whose components are left Archimedean semigroups and nil-extensions of left groups were studied by Bogdanović and Ćirić in [**4**], 1995, and Shevrin in [**15**], 1994, respectively. Also, some other kinds of semilattice-matrix decompositions were studied by Chu, Guo and Ren in [**6**], 1989.

It is important to note that semilattice-matrix decompositions are more general than many other significant kinds of decompositions of semigroups. For example, it is evident that semilattice and matrix

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