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u-INDEPENDENCE AND QUADRATIC u-INDEPENDENCE IN THE CONSTRUCTION OF INDECOMPOSABLE FINITELY GENERATED MODULES

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ABSTRACT. Let R be a valuation domain having an ideal I such that a maximal immediate extension S of R contains four units u-independent over I. We construct a 4-generated indecomposable R-module M with Goldie dimension $g(M) = 2$. We thus supplement a result by Lunsford who constructed indecomposable finitely generated R-modules making use of sets of quadratically u-independent elements of S.

1. Introduction. Let R be a valuation domain, and let S be a fixed maximal immediate extension of R. There is a somewhat standard way to define finitely generated R-modules M by generators and relations, relating M to a set of units $u_1, \dots, u_n$ of S. Starting with [**5**] and [**8**], an extensive use of this idea was made. See also the books by Fuchs and Salce [**2**, Chapter 9] and [**3**, Chapter 5]. The notion of u-independence of units $u_1, \dots, u_n$ of S over an ideal I of R was introduced in [**8**] and investigated further in [**9**]. It was used to show the existence of indecomposable finitely generated R-modules M (related with $u_1, \dots, u_n$) with minimal number of generators $l(M) = n + 1$ and Goldie dimension $g(M) = n$. This solved the problem of finding indecomposable finitely generated R-modules with Goldie dimension greater than one. However, it is worth noting that the argument developed in [**8**] worked only in the case when $l(M) = g(M) + 1$.

Lunsford [**4**] in 1995 gave a natural generalization of u-independence, defining *quadratic u-independence* of units $u_1, \dots, u_n$ of S over an ideal I. Starting with a sufficiently large set of units of S, for any pair of positive integers h, k he defined by generators and relations an R-module M with $l(M) = h + k$ and $g(M) = h$. Actually this type of module had already been introduced in 1987 by Salce and Zanardo [**7**]. Using quadratic u-independence, Lunsford was able to prove that such

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