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CHAIN CATEGORIES OF MODULES AND SUBPROJECTIVE REPRESENTATIONS OF POSETS OVER UNISERIAL ALGEBRAS

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ABSTRACT. Filtered chain categories $\mathcal{C}(s, R)$ of modules over a commutative artinian uniserial ring R and their representation types are studied in the paper. A tame-wild dichotomy theorem is proved in case R is a finite dimensional K-algebra over an algebraically closed field K. The pairs (s, R) for which $\mathcal{C}(s, R)$ is of finite representation type are determined. In case $R = K[t]/(t^m)$ and K is algebraically closed, the pairs (s, m) for which $\mathcal{C}(s, R)$ is of tame representation type are listed. The problem is reduced to the study of categories of subprojective representations of posets over uniserial algebras and then to representations of posets over a field by applying a Galois covering functor technique.

1. Introduction. Let R be a unitary commutative artinian uniserial ring with the Jacobson radical $J(R)$. We recall that R is uniserial if the ideals of R form a finite chain. In this case $J(R)$ is the unique maximal ideal of R, and there is an integer $m \geq 1$ such that $J(R)^m = 0$, $J(R)^{m-1} \neq 0$ and any ideal of R appears in the chain

$$R \supset J(R) \supset J(R)^2 \supset \cdots \supset J(R)^{m-1} \supset J(R)^m = 0. \tag{1.1}$$

Examples of such rings R are the ring $\mathbf{Z}/p^m\mathbf{Z}$ of integers modulo p^m or the uniserial K-algebra $F_m = K[t]/(t^m)$, where $p \geq 2$ is a prime, $m \geq 1$ is an integer and K is a field.

Following Arnold [**1**] and [**2**], given an integer $s \geq 1$ we consider the filtered chain category $\mathcal{C}(s, R)$ whose objects are filtered s-chains

$$C = (C_1 \subseteq C_2 \subseteq \cdots \subseteq C_{s-1} \subseteq C_s) \tag{1.2}$$

of finitely generated R-modules $C_1, \ldots, C_s$, and a morphism from C to C' in $\mathcal{C}(s, R)$ is an R-module homomorphism $f : C_s \to C'_s$ such that

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