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PRE-ABELIAN CLAN CATEGORIES

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ABSTRACT. Categories of representations of clans without special loops, and with a linear ordering at each vertex, are studied with an eye toward identifying those that have kernels and cokernels. A complete characterization is given for simple graphs whose vertices have degree at most two.

1. Representations of clans. I'm not going to give the definition of an arbitrary clan [**1**], but only a very restricted version which will cover the cases I want to look at here. A (linear ordinary) *clan* consists of

- A finite graph, possibly with multiple edges and loops,
- At each vertex v an enumeration $e(v,1),\dots,e(v,d)$ of the edges incident to v in which each incident loop appears twice and the other edges appear once. The integer d is the *degree* of the vertex.

We say that an edge e *joins* (v,i) *with* (w,j) *if* $e=e(v,i)=e(w,j)$ and $(v,i)\neq(w,j)$.

In contrast to the general notion of a clan, no field is mentioned because we don't allow "special loops." Representations of clans decompose canonically into representations of their components, so we may assume that the graph is connected. As in [**1**], we will assume that there are no vertices of degree 0 which, for a connected graph, simply says that it has an edge.

If k is a field, then a *k-representation* M of a clan associates a finite-dimensional vector space $M(v)$ over k to each vertex v of the clan, together with a filtration

$$0=M(v)_0\subset M(v)_1\subset\cdots\subset M(v)_{d(v)}=M(v)$$

of $M(v)$ where $d(v)$ is the degree of v. Moreover, if the edge e joins (v,i) with (w,j), then M associates with e an isomorphism M_e between $M(v_i)/M(v)_{i-1}$ and $M(w)_j/M(w)_{j-1}$.

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