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ISOTYPE WARFIELD SUBGROUPS OF GLOBAL WARFIELD GROUPS

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ABSTRACT. Using a new characterization of global Warfield groups, necessary and sufficient conditions are given for an isotype subgroup of a global Warfield group to be itself, Warfield. Our result generalizes similar theorems in the simpler contexts of totally projective p-groups and p-local Warfield groups.

1. Introduction. We shall be dealing exclusively with additively written abelian groups, hereafter referred to simply as "groups," and G will always denote such a group. We emphasize from the outset that G is allowed to be mixed.

Recall that a group is *simply presented* if it can be presented by generators and relations where each relation involves at most two generators. In the torsion and torsion-free settings, a summand of a simply presented group is again simply presented. However, for mixed groups G this is not generally the case. By definition, a *global Warfield group* is a direct summand of a simply presented group. Most of the early theory of global Warfield groups was developed by Hunter, Richman and Walker [**8, 9, 10**]. But it was not until the introduction of knice subgroups [**3**] and the attainment of an Axiom 3 characterization [**4**] that fundamental problems regarding isotype subgroups became accessible (for prime examples, see [**4**] and [**11**]).

In this paper we again demonstrate the power of the theory of knice subgroups and Axiom 3 by finding necessary and sufficient conditions for an isotype subgroup of a global Warfield group to be itself Warfield. Our result generalizes the earlier treatments of isotype subgroups of totally projective p-groups in [**2**], and of p-local Warfield groups in [**7**]. To a certain extent, our theorem and proof are modeled after the special case in [**7**]; however, the generalization from the local to

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