

SHARP TYPES REVISITED

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Dedicated to Jim Reid on the occasion of his retirement

ABSTRACT. A generalized and canonical definition of “sharp type” is given and a decomposition theorem is proved for arbitrary almost completely decomposable groups. As an application we show that an almost completely decomposable group whose critical typeset is a garland is a direct sum of rank-one and rank-two groups.

1. Introduction. An *almost completely decomposable group* X is a finite (torsion-free abelian) extension of a completely decomposable group A of finite rank. In 1974, Lady [4] initiated a systematic theory of such groups based on the fundamental concept of *regulating subgroup*. The regulating subgroups can be defined as the completely decomposable subgroups of least index in an almost completely decomposable group X . This least index is the *regulating index* $\text{rgi } X$. An almost completely decomposable group is *local* if its regulating index is a prime power; otherwise it is a *global* group. An accessible class of almost completely decomposable groups with arbitrary critical typeset is the class of crq-groups, namely those groups X containing a completely decomposable subgroup A such that X/A is a finite cyclic group. Campagna [2] showed that this is equivalent to the existence of a regulating subgroup A such that X/A is cyclic. The local and global crq-groups have been studied in [7] and [3]. For local crq-groups the direct decompositions with indecomposable summands were completely determined in [7]. The concept of “sharp type” was essentially involved in this determination and in [3] “sharp type” was defined for global crq-groups and a decomposition theorem was proved. Both definitions of sharp type were based on special representations of crq-groups. In [6] it was demonstrated that crq-groups are largely determined by the invariants $\text{rgi } X[\tau]$ (see Theorem 3.1). This leads to a generalized concept

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