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THE KRULL-SCHMIDT PROPERTY FOR IDEALS AND MODULES OVER INTEGRAL DOMAINS

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Dedicated to Jim Reid

ABSTRACT. We examine when versions of the Krull-Schmidt property hold for (1) direct sums of ideals of integral domains, (2) direct sums of indecomposable submodules of finitely generated free modules, and (3) direct sums of rank one torsion-free modules. Our main results are formulated for modules over h-local integral domains without recourse to finite generation for the modules. This leads to some new results for Krull-Schmidt properties of modules over Noetherian and Prüfer domains.

1. Introduction. Let R be a commutative integral domain and $\mathcal{C}$ a class of R-modules. The *Krull-Schmidt property* holds for $\mathcal{C}$ if, whenever

$$G_1 \oplus G_2 \oplus \cdots \oplus G_n \cong H_1 \oplus H_2 \oplus \cdots \oplus H_m$$

for $G_i, H_j \in \mathcal{C}$, then $n = m$ and, after reindexing, $G_i \cong H_i$ for all $i \leq n$. If, instead of $G_i \cong H_i$, we require only that $k > 0$ exists such that $G_i^{(k)} \cong H_i^{(k)}$ for all i, then we say the *weak Krull-Schmidt property* holds for $\mathcal{C}$. (We write $G^{(k)}$ for a direct sum of k copies of a module G.)

In this article we examine Krull-Schmidt properties for certain classes of indecomposable torsion-free modules over commutative integral domains. By a *torsionless* module over a domain R, we mean a submodule of a finitely generated free R-module. An integral domain R has the *torsion-free Krull-Schmidt property, TFKS*, if the class of indecomposable torsionless R-modules has the Krull-Schmidt property; R has *weak TFKS* if this class has the weak Krull-Schmidt property.

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