

COMPLETELY DECOMPOSABLE ABELIAN GROUPS WITH A DISTINGUISHED CD SUBGROUP

MANFRED DUGAS AND K.M. RANGASWAMY

ABSTRACT. We define a category $CD^1(T, p)$, p a prime and T a set of types, consisting of all pairs $V = (C, D)$ where C is a completely decomposable group with critical type set T and D a completely decomposable subgroup with $p^e C \subseteq D \subseteq C$ for some $e \geq 1$. We show that while indecomposables in this category have rank at most one if T is an antichain, we observe “wild” behavior if T contains comparable elements.

I. Introduction. One of the true chestnuts of the theory of abelian groups is the stacked basis theorem that goes back to work of C.F. Gauss [4] which can be stated as follows: Let F be a free module of finite rank over a principal ideal domain S and X a submodule of F . Then F has a basis $B = \{b_1, \dots, b_n\}$ such that $X = \bigoplus_{i=1}^n b_i s_i S$ for some $s_i \in S$ with $s_i \in s_{i-1} S$, $1 \leq i \leq n$. This implies that F/X is a direct sum of cyclic S -modules. We refer to [3] for a discussion of the history and references for this result. More recently, Benabdallah and Ould-Beddi [2] proved a version of the stacked basis theorem for homogeneous completely decomposable (hcd) groups of finite rank, which was extended by Ould-Beddi and Strüngmann [5] to the infinite rank case: Let C be an hcd group and X a (hcd) subgroup of C of bounded index. Then C has a stacked basis B for X , i.e., $C = \bigoplus_{b \in B} \langle b \rangle_*^C$ and $X = \bigoplus_{b \in B} \langle s_b b \rangle_*^X$ for some $s_b \in \mathbf{Z}$, where $\langle b \rangle_*^X$ denotes the purification of the subgroup generated by b inside X . This makes it natural to ask about the case where C is not hcd but completely decomposable (cd) of finite rank with critical type set T .

It is easy to see that the same result holds in this case if T is an antichain but, as we shall see, things are dramatically different if the critical type set T contains comparable elements. This note attempts

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