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GENERALIZED TRIANGULAR MATRIX RINGS AND THE FULLY INVARIANT EXTENDING PROPERTY

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ABSTRACT. A module M is called (*strongly*) *FI-extending* if every fully invariant submodule of M is essential in a (fully invariant) direct summand of M. A ring R with unity is called *quasi-Baer* if the right annihilator of every ideal is generated, as a right ideal, by an idempotent. For semi-prime rings the FI-extending condition, strongly FI-extending condition and quasi-Baer condition are equivalent. In this paper we fully characterize the 2-by-2 generalized (or formal) triangular matrix rings which are either (right) FI-extending, (right) strongly FI-extending, or quasi-Baer. Examples are provided to illustrate and delimit our results.

0. Introduction. All rings are associative with unity and all modules are unital. Throughout this paper T will denote a 2-by-2 generalized (or formal) triangular matrix ring

$$\begin{pmatrix} S & M \\ 0 & R \end{pmatrix},$$

where R and S are rings and M is an (S, R)-bimodule.

Generalized triangular matrix rings have proven to be extremely useful in ring theory. They provide a good source of examples and counterexamples, e.g., see [**11**, pp. 46–48 and 79–80] and [**10**], as well as providing a framework to explore the connections between End (M_R), M and R when $S = \text{End}\,(M_R)$.

Recently several aspects of injectivity and projectivity in the context of generalized triangular matrix rings have been investigated by Haghany-Varadarajan [**8, 9**] and Tercan [**13**]. Tercan was able to obtain a characterization of the right nonsingular right extending (or CS)

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