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# COMPARISON THEOREMS AND STRONG OSCILLATION IN THE HALF-LINEAR DISCRETE OSCILLATION THEORY

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ABSTRACT. Consider the second order half-linear difference equation

$$\text{(HL)} \qquad \triangle(r_k|\triangle y_k|^{\alpha-1}\operatorname{sgn}\triangle y_k) + p_k|y_{k+1}|^{\alpha-1}\operatorname{sgn} y_{k+1} = 0, \quad \alpha > 1.$$

In the first part we give various types of comparison theorems for this equation, including the so-called telescoping principle, and also for the associated generalized Riccati difference equation. In the second part, we present criteria for strong (non)-oscillation of (HL) and related results. The paper is finished by an example where oscillatory properties of a generalized discrete Euler equation are investigated.

**1. Introduction.** This paper is a further demonstration of the fact that one can extend (in a sense of the "half-linear generalization" and of the discretization) the most results of the oscillation theory of the Sturm-Liouville linear differential equation

$$(r(t)y')' + p(t)y = 0$$

to the half-linear difference equation

$$\triangle(r_k\Phi(\triangle y_k)) + p_k\Phi(y_{k+1}) = 0, \tag{1}$$

where $r_k, p_k$ are real-valued sequences defined on $\mathbf{N}$ with $r_k \neq 0$ and $\Phi(y) := |y|^{\alpha-1}\operatorname{sgn} y = |y|^{\alpha-2}y$ with $\alpha > 1$. Since the Sturm type

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