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NESTED SEQUENCES OF BALLS, UNIQUENESS OF HAHN-BANACH EXTENSIONS AND THE VLASOV PROPERTY

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ABSTRACT. In this work we characterize when a single linear functional dominated by a sublinear functional p on a subspace of a real vector space has a unique extension to the whole space dominated by p in terms of nested sequences of "p-balls" in a quotient space. This is then specialized to obtain characterizations of the phenomenon when a single linear functional on a subspace of a Banach space has unique norm-preserving extension to the whole space, thus localizing and generalizing some recent work of Oja and Põldvere. These results are used to characterize w^*-asymptotic norming properties in terms of nested sequences of balls in X extending the notion of Property (V) introduced by Sullivan. A variety of examples and applications of the main results are also presented.

1. Introduction. We work with *real* scalars. For a Banach space X, we denote by $B(X), S(X)$ and $B(x,r)$, or $B[x,r]$, respectively, the closed unit ball, the unit sphere and the open, or closed, ball of radius $r > 0$ around $x \in X$. When X is just a vector space, we will denote linear functionals on X by f, g, etc., while for a Banach space X, elements of the dual X^* will be denoted by x^*, y^*, etc.

Definition 1.1. A closed subspace Y of a Banach space X is said to be a U-subspace of X if for any $y^* \in Y^*$ there exists a unique Hahn-Banach (i.e., norm-preserving) extension of y^* in X^*.

X is said to be Hahn-Banach smooth if X is a U-subspace of X^{**} under the canonical embedding of X in X^{**}.

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