

## HERMITE ORTHOGONAL RATIONAL FUNCTIONS

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Dedicated to William B. Jones on the occasion of his 70th birthday

**ABSTRACT.** We recount previous development of  $d$ -fold doubling of orthogonal polynomial sequences and give new results on rational function coefficients, recurrence formulas, continued fractions, Rodrigues' type formulas, and differential equations, for the general case and, in particular, for the  $d$ -fold Hermite orthogonal rational functions.

**1. Introduction.** Orthogonal rational functions and related subjects today are active areas of investigation for researchers around the world, yielding theoretical and applicable results spanning a gamut of interests. Included among these are various techniques and methods of numerical integration. [1, 2, 3, 6, 9, 10, 11, 15] are examples. It was shown in [10] that the transformation given in [12, 13] taking systems of orthogonal polynomials to systems of orthogonal Laurent polynomials applies in a more general context of certain function spaces, leading to a recursive construction of hierarchies of rational function spaces. Consequently, Gaussian quadrature rules of a new kind were obtained, ones in which the number of nodes are doubled and redoubled and the abscissas and weights themselves are given by simple recursive formulas, extending the results in [9, 11].

It is our goal here to explicate the family of orthogonal rational function sequences that is constructed from the classical Hermite polynomials, although new and recounted theorems are given for the general case of  $d$ -fold doubling of orthogonal polynomial sequences that was introduced in [10].

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1991 AMS *Mathematics Subject Classification.* Primary 65D32, Secondary 26C15.

*Key words and phrases.* Orthogonal rational functions.

Received by the editors on September 1, 2001, and in revised form on February 18, 2002.

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