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CONTINUED FRACTIONS, WAVELET TIME OPERATORS, AND INVERSE PROBLEMS

KARL GUSTAFSON

ABSTRACT. The spectral properties of the analytic theory of continued fractions, orthogonal functions, and rational approximation, are examined from the point of view of operator theory. New historical and mathematical perspectives are provided. Then a general question is posed: what spectral information is needed to uniquely determine an operator? The time operator induced by an arbitrary wavelet basis is presented as an example. This question is then directed at continued fractions.

1. Introduction. This paper, invited for the *Conference on the analytic theory of continued fractions, orthogonal functions, rational approximation and related topics* in honor of William B. Jones's 70th birthday, will focus on the related topic of *operator theory* and how this fourth topic relates to the first three topics. This investigation grew out of a conversation between this author and Professor Jones a few months prior to the conference. In that conversation I asked, "what is the spectral theory (of the first three topics)?" and a few days later I found the recent paper [**27**] in my mailbox. That paper provides an excellent overview of orthogonal Laurent polynomials, moment theory, continued fractions, Gaussian quadrature, Stieltjes transforms, and linear functionals. As such, [**27**] provides a summary of results of 'spectral' type as seen from the viewpoint of the *continued fraction community* (for convenience and with apologies I will use this term to represent all three principal topics of this conference).

The first goal of this paper is to "answer my own question," to wit: to supplement the viewpoint of [**27**], and some of the extensive literature which it represents, by my own commentary here, as a representative of the *operator theory community.* Of course, to fully answer my question would be a lifetime's work, so here I will only be able to patch the two

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