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CONVERGENCE OF PPC-CONTINUED FRACTION APPROXIMANTS IN FREQUENCY ANALYSIS

WILLIAM B. JONES, VIGDIS BREVIK PETERSEN
AND HAAKON WAADELAND

Dedicated to the memory of Wolfgang J. Thron
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1. Introduction. Many natural phenomena can be represented by real-valued functions of the form

$$G(t) = \sum_{j=-I}^{I} \alpha_j e^{i2\pi f_j t}, \quad I \in \mathbf{N}, \tag{1.1a}$$

where t denotes time (sec.), the *frequencies* f_j are in cycles per sec (Hertz) and the *complex amplitudes* α_j satisfy

$$\alpha_0 \geq 0 \neq \alpha_j = \bar{\alpha}_{-j}, \quad f_j = -f_{-j}, \quad \text{for } j = 1, 2, \dots, I \tag{1.1b}$$

and

$$0 = f_0 < f_1 < f_2 < \cdots < f_I. \tag{1.1c}$$

The *frequency analysis problem* (FAP) consists of determining the unknown frequencies f_j by using N values of "observed data"

$$G(t_m), \quad m = 0, 1, \dots, N-1, \quad \text{where } t_m := m\Delta t, \quad \Delta t > 0. \tag{1.2}$$

For convenience we introduce *normalized frequencies*

$$\omega_j := 2\pi f_j \Delta t, \quad j = 0, \pm 1, \pm 2, \dots, \pm I, \tag{1.3a}$$

with the restrictions imposed by

$$0 = \omega_0 < \omega_1 < \omega_2 < \cdots < \omega_I < \pi \tag{1.3b}$$

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