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STRONG ASYMPTOTICS FOR RELATIVISTIC HERMITE POLYNOMIALS

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ABSTRACT. Strong asymptotic results for relativistic Hermite polynomials $H_n^N(z)$ are established as $n, N \to \infty$, for the cases where $N = an + \alpha + 1/2$, $a \geq 0, \alpha > -1$, or $N/n \to \infty$, thereby supplementing recent results on weak asymptotics for these polynomials. Depending on growth properties of the ratio N/n for the rescaled polynomials $H_n^N(c_n z)$ (c_n being suitable positive numbers, $n, N \to \infty$), formulae of Plancherel-Rotach type are derived on the oscillatory interval, in the complex plane away from the oscillatory region, and near the endpoints of the oscillatory interval.

1. Introduction and summary. In this paper we continue the study of asymptotic properties of relativistic Hermite polynomials H_n^N. This set of polynomials has been introduced for the investigation of the harmonic oscillator in the frame of relativistic quantum theory [**1**]. Here n denotes the principal quantum number, being a nonnegative integer, and N is a positive parameter describing the underlying relativistic effect such that the system approaches the classical (nonrelativistic) model as $N \to \infty$. This transition is made precise by the limit relation

$$\lim_{N\to\infty} H_n^N(z) = H_n(z), \quad z \in \mathbf{C}, \quad n \in \mathbf{N}_0, \tag{1.1}$$

where H_n denotes the well-known Hermite polynomials [**17**, Chapter V]. Similar to H_n its relativistic counterpart can be characterized by several properties, for instance as polynomial solution of the second order linear differential equation [**1, 11, 15, 19, 20**]

$$\left(1 + \frac{z^2}{N}\right)y'' - \frac{2}{N}(N + n - 1)zy' + \frac{n}{N}(2N + n - 1)y = 0, \tag{1.2}$$

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