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ENTIRE FUNCTIONS OF EXPONENTIAL TYPE AND UNIQUENESS CONDITIONS ON THEIR REAL PART

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ABSTRACT. This paper extends uniqueness results due to Boas and Trembinska, on entire functions with exponential growth whose real part vanishes on lattice points. Here the case is studied where the real part of the function satisfies given relations or assumes prescribed values at lattice points. These results are obtained thanks to such tools as analytic functionals, their Fourier-Borel transform and some operators acting in the space of entire functions in $\mathbf{C}^N$ of exponential type, including difference and differential operators of infinite order with constant coefficients. There are also applications to some difference equation studied by Buck, Boas and Yoshino.

0. Introduction.

Carlson's theorem for entire functions in $\mathbf{C}$ of exponential type $< \pi$ gives rise to various generalizations. It involves entire functions f such that

$$|f(z)| \leq Ce^{\tau|z|}, \quad \text{for each } z \in \mathbf{C} \tag{1}$$

where $C > 0$ and $0 < \tau < \pi$ are two constants. This theorem states that such a function f is identically zero in $\mathbf{C}$ as soon as $f(n) = 0$ for each $n \in \mathbf{N}$ (see [**7, 21**]). This uniqueness theorem extends to entire functions in $\mathbf{C}^N$ (see [**4, 16**]) and to harmonic functions in $\mathbf{R}^N$ (see [**2**]), which vanish on $\mathbf{N}^N$ and grow exponentially.

In the case $N = 1$, [**8**] studies the following situation:

Theorem I [**8**]. *A function f entire in $\mathbf{C}$, of exponential type $< \pi$, whose real part vanishes on $\mathbf{Z}$ and $\mathbf{Z} + i$, is constant: $f \equiv ib$, $b \in \mathbf{R}$.*

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