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THE DISCRIMINANT OF A CYCLIC FIELD OF ODD PRIME DEGREE

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ABSTRACT. Let p be an odd prime. Let $f(x) \in \mathbf{Z}[x]$ be a defining polynomial for a cyclic extension field K of the rational number field $\mathbf{Q}$ with $[K:\mathbf{Q}] = p$. An explicit formula for the discriminant $d(K)$ of K is given in terms of the coefficients of $f(x)$.

1. Introduction. Throughout this paper p denotes an odd prime. Let K be a cyclic extension field of the rational field $\mathbf{Q}$ with $[K:\mathbf{Q}] = p$. In this paper we give an explicit formula for the discriminant $d(K)$ of K in terms of the coefficients of a defining polynomial for K. We prove

Theorem 1. *Let* $f(X) = X^p + a_{p-2}X^{p-2} + \cdots + a_1 X + a_0 \in \mathbf{Z}[X]$ *be such that*

$$\mathrm{Gal}\,(f) \simeq \mathbf{Z}/p\mathbf{Z} \tag{1}$$

and

(2) *there does not exist a prime* q *such that*

$$q^{p-i} | a_i, \quad i = 0, 1, \ldots, p-2.$$

Let $\theta \in \mathbf{C}$ *be a root of* $f(X)$ *and set* $K = \mathbf{Q}(\theta)$ *so that* K *is a cyclic extension of* $\mathbf{Q}$ *with* $[K:\mathbf{Q}] = p$. *Then*

$$d(K) = f(K)^{p-1}, \tag{3}$$

where the conductor $f(K)$ *of* K *is given by*

$$f(K) = p^{\alpha} \prod_{\substack{q \equiv 1 \ (\mathrm{mod}\ p) \\ q | a_i, i=0,1,\ldots,p-2}} q, \tag{4}$$

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