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ON THE COMMUTANT OF MULTIPLICATION OPERATORS WITH ANALYTIC SYMBOLS

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ABSTRACT. Let $\mathcal{B}$ be a certain Banach space consisting of analytic functions defined on a bounded domain G in the complex plane. Let $\phi \in \mathcal{B}$ be a function which is analytic on G and continuous on $\overline{G}$. Assume that M_ϕ denotes the operator of multiplication by ϕ. We characterize the commutant of M_ϕ that is the set of all bounded operators T such that $M_\phi T = TM_\phi$. Under certain conditions on ϕ, we show that $T = M_\varphi$ for some function φ in $\mathcal{B}$.

1. Introduction. Let $\mathcal{B}$ be a Banach space consisting of analytic functions defined on a bounded domain G in the complex plane such that $\mathcal{B}$ satisfies conditions a, b, c, d as follows:

(a) $1 \in \mathcal{B}$, $z\mathcal{B} \subset B$.

(b) For every $\lambda \in G$ the evaluation functional at λ, $e_\lambda : \mathcal{B} \to \mathbf{C}$, given by $f \mapsto f(\lambda)$, is bounded.

(c) $\operatorname{ran}(M_z - \lambda) = \ker e_\lambda$ for every $\lambda \in G$.

(d) If $f \in \mathcal{B}$ and $|f(\lambda)| > c > 0$ for every $\lambda \in G$, then $1/f$ is a multiplier of $\mathcal{B}$.

Throughout this article by a Banach space of analytic functions $\mathcal{B}$ on G we mean one satisfying the above conditions.

Some examples of such spaces are as follows:

1) The algebra $A(G)$ which is the algebra of all continuous functions on the closure of G that are analytic on G.

2) The Bergman space of analytic functions defined on G, $L_a^p(G)$ for $1 \le p \le \infty$.

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