ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 33, Number 3, Fall 2003

EXTREMAL BOUNDED SLIT MAPPINGS FOR LINEAR FUNCTIONALS

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ABSTRACT. Let $S(M)$ be the class of holomorphic univalent functions $f(z) = z + a_2 z^2 + \dots$, $|f(z)| < M$, $|z| < 1$ and $L(f) = \sum_{k=2}^{n} \lambda_k a_k$, $(\lambda_2, \dots, \lambda_n) \in \mathbf{R}^{n-1}$. We prove that under some conditions among all bounded slit mappings only the Pick functions can be extremal for $\Re L(f)$ in $S(M)$ provided M is close to 1. In particular, if $\alpha > 0$, $(n-1)$ and $(m-1)$ are odd and relatively prime, then the Pick function maximizes $\Re(a_n + \alpha a_m)$ in $S(M)$ for M close to 1.

1. Introduction. Let $S(M)$, $M > 1$, be the class of holomorphic functions f in the unit disk $D = \{z : |z| < 1\}$,

$$f(z) = z + a_2 z^2 + \dots, \quad z \in D,$$

which are univalent and bounded by M in D, i.e., $|f(z)| < M$, $z \in D$.

Denote by $S^1(M)$ the class of functions $f \in S(M)$ which map D onto the disk D_M of radius M centered at the origin and slit along an analytic curve. An important member of $S^1(M)$ is the so-called Pick function $P_M(z)$ which maps D onto D_M slit along the segment $[-M, -M(2M - 1 - 2\sqrt{M(M-1)})]$.

Consider a linear continuous functional L on $S(M)$ given by

$$L(f) = \sum_{k=2}^{n} \bar{\lambda}_k a_k, \quad \lambda_k \in \mathbf{C}, \quad k = 2, \dots, n.$$

So L is determined by the vector $\boldsymbol{\lambda} = (\lambda_2, \dots, \lambda_n) \in \mathbf{C}^{n-1}$.

We will prove the following

Theorem 1. *Let* $\boldsymbol{\lambda} = (\lambda_2, \dots, \lambda_n) \in \mathbf{R}^{n-1}$ *and*

$$\max_{f \in S(M)} \Re L(f) = \Re L(f_0)$$

Received by the editors on November 10, 2000.