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FIRST-COUNTABILITY, SEQUENTIALITY AND TIGHTNESS OF THE UPPER KURATOWSKI CONVERGENCE

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ABSTRACT. The first-countability of the upper Kuratowski convergence is characterized in terms of the underlying convergence. If we start with a topology, first-countability is also equivalent to countable tightness of the upper Kuratowski topology, equivalently upper Kuratowski convergence. This result is applied to consonance and its analogous for the real-valued continuous dual is applied to a generalized (real-valued) consonance result.

1. Introduction. Relations between topological properties of a space X and the corresponding properties of function spaces on X, and in particular those of various hyperspace topologies on its closed sets, have been intensively investigated ([**26, 30, 6, 5, 16**] among a lot of others). In such studies a great collection of hyperspace structures emerged. Moreover, several non topological structures appear naturally ([**32, 35, 7**]). The upper Kuratowski convergence plays a particular role in the lattice of all the convergence structures one can endow a hyperspace with. Indeed, this is the least convergence that makes the evaluation (by identifying closed sets with their indicator functions) jointly continuous. This least convergence, transposed on the lattice of open sets, is homeomorphic to the Scott convergence, classically used and studied since its introduction by Scott in [**42**] in the extensive literature on lattice theory and continuous lattices, e.g., [**28, 29, 27, 11**]. In the sublattice of hyperspace topological structures, studied for instance in [**13**] and [**14**], there is in general no such least structure available. However, the topological modification of the upper Kuratowski convergence, called upper Kuratowski topology (homeomorphically Scott topology), retains some of the nice properties of the upper Kuratowski convergence and turns out to be of particular interest. Hence, even if you focus your attention on topological hyperspace structures and, in

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