

ASYMPTOTIC BEHAVIOR AND OSCILLATION OF DELAY PARTIAL DIFFERENCE EQUATIONS WITH POSITIVE AND NEGATIVE COEFFICIENTS

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ABSTRACT. We obtain sufficient conditions for the oscillation of all solutions of the linear partial difference equations with positive and negative coefficients of the form

$$A_{m-1,n} + A_{m,n-1} - A_{mn} + pA_{m+k} - qA_{m+k'} = 0,$$

$n+l \qquad n+l'$

and

$$A_{m-1,n} + A_{m,n-1} - A_{mn} + p_{mn}A_{m+k} - q_{mn}A_{m+k'} = 0,$$

$n+l \qquad n+l'$

where $m, n = 0, 1, \dots$, and k, k', l', l are nonnegative integers $p, q \in (0, \infty)$, and coefficients $\{q_{mn}\}$ and $\{p_{mn}\}$ are sequences of nonnegative real numbers. In this paper $A_m = A_{m,n}$.

1. Introduction. Partial difference equations arise from various practical problems and numerical analysis of partial difference equations [1-2]. In this area, the oscillatory and nonoscillatory behaviors of delay partial difference equations have been investigated in, for example, [3, 4, 6-11].

In this paper we consider the linear partial difference equations with positive and negative coefficients in the form

$$(1.1) \quad A_{m-1,n} + A_{m,n-1} - A_{mn} + pA_{m+k} - qA_{m+k'} = 0,$$

$n+l \qquad n+l'$

and

$$(1.2) \quad A_{m-1,n} + A_{m,n-1} - A_{mn} + p_{mn}A_{m+k} - q_{mn}A_{m+k'} = 0.$$

$n+l \qquad n+l'$

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