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OSCILLATION CRITERIA FOR SECOND-ORDER HALF-LINEAR ORDINARY DIFFERENTIAL EQUATIONS WITH DAMPING

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Dedicated to Professor Wenyuan Chen on his seventieth birthday

ABSTRACT. By using averaging functions and an inequality due to Hardy, Littlewood and Polya, several new oscillation criteria are established for the half-linear damped differential equation

$$[r(t)|y'(t)|^{\alpha-1}y'(t)]' + p(t)|y'(t)|^{\alpha-1}y'(t) + q(t)|y(t)|^{\alpha-1}y(t) = 0,$$

where $r \in C^1([t_0,\infty);(0,\infty))$, $\alpha > 0$ and $p, q \in C[t_0,\infty)$. Our results extend and improve the oscillation criteria of Kamenev, Li and Philos for linear equations. Several examples are inserted in the text to illustrate our results.

1. Introduction. In this paper we consider the problem of oscillation of the second-order half-linear damped differential equation

$$[r(t)|y'(t)|^{\alpha-1}y'(t)]' + p(t)|y'(t)|^{\alpha-1}y'(t) + q(t)|y(t)|^{\alpha-1}y(t) = 0, \tag{1.1}$$

on the half-line $[t_0,\infty)$. In equation (1.1) we assume that $r \in C^1([t_0,\infty);(0,\infty))$, $p, q \in C[t_0,\infty)$ and $\alpha > 0$ is a constant.

We recall that a function $y : [t_0,t_1) \to (-\infty,\infty)$, $t_1 > t_0$ is called a solution of equation (1.1) if $y(t)$ satisfies equation (1.1) for all $t \in [t_0,t_1)$. In the sequel it will always be assumed that solutions

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