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ON SOME PROPERTIES OF DEDDENS ALGEBRAS

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ABSTRACT. Deddens algebras and Shulman subspaces are introduced and their properties are studied. The descriptions of Deddens algebras associated with nilpotent and idempotent elements are given.

1. Introduction. Let H be a Hilbert space, $B(H)$ be an algebra of all bounded linear operators in H. In [**1**], Deddens determined for any invertible operator A from $B(H)$ the following algebra:

$$B_A \overset{\text{def}}{=} \Big\{X \in B(H) : \sup_{n\geq 0} \Big\|A^n X A^{-n}\Big\| \overset{\text{def}}{=} C_X < +\infty\Big\}.$$

It was proved in [**1**] that, for $A \geq 0$, B_A coincides with the nest algebra generated by the nest $\{E_A([0,\lambda]) : \lambda \geq 0\}$ (where E_A is the spectral measure of A) that gives a suitable characterization of nest algebras in all respects. Recently Todorov [**7**] has extended this result to weakly or strongly closed bimodules of a nest algebra. In [**2**] Deddens and Wong have proved that if $A = \lambda I + N$, where $\lambda \in \mathbf{C} \setminus \{0\}$ is a complex number, and N is a nilpotent operator, then the algebra B_A coincides with the commutant $\{A\}'$ of A. In their proof of the last statement the Hilbert property of the space H is essentially used.

The main aim of this paper is to show that the result of Deddens and Wong is valid in any unital Banach algebra.

2. Deddens algebras. Let B be a Banach algebra with the unit e. For any invertible element $a \in B$ put

$$B_a \overset{\text{def}}{=} \Big\{x \in B : \sup_{n\geq 0} \|a^n x a^{-n}\| \overset{\text{def}}{=} C_x < +\infty\Big\}.$$

We call the algebra B_a the Deddens algebra.

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