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THE ASYMPTOTIC OF SOLUTIONS FOR A CLASS OF DELAY DIFFERENTIAL EQUATIONS

JAN ČERMÁK

ABSTRACT. We study the asymptotic properties of solutions of the differential equation

$$\dot{x}(t) = -c(t)[x(t) - Lx(\tau(t))]$$

with a positive continuous function $c(t)$, a nonzero real constant L and unbounded lag. We establish conditions under which each solution of this equation approaches a solution of the auxiliary functional equation

$$\psi(t) = L\psi(\tau(t)).$$

Moreover, we investigate some modifications of the studied equation and give comparisons with the known results.

1. Introduction. We investigate the asymptotic behavior of solutions of the delay differential equation

$$\dot{x}(t) = -c(t)[x(t) - Lx(\tau(t))], \quad t \in I = [t_0, \infty), \tag{1.1}$$

where $c(t)$ is a positive continuous function on I, L is a nonzero real scalar and $\tau(t)$ is a continuously differentiable function such that $\tau(t) \to \infty$ as $t \to \infty$, $\tau(t) < t$ and $0 < \dot{\tau}(t) \le \lambda < 1$ for every $t \in I$.

Our assumptions imply that the lag $t - \tau(t)$ must be necessarily unbounded on I. Equations with this type of a delay have diverse applications in areas ranging from the number theory to industrial problems. The objective of many authors have been especially equations with the proportional argument (see [**6**], [**8**], [**10**], [**14**] and others) and equations with the linearly transformed argument (see [**3**] and references

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