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UNIQUE SOLVABILITY OF AN ORDINARY FREE BOUNDARY PROBLEM

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ABSTRACT. In this paper an existence theorem for an ordinary free boundary problem is given. The result is based on differential inequalities and completes the uniqueness theorem by R.C. Thompson in 1982.

1. Introduction. In the present paper we consider the ordinary free boundary problem:

Find $s > 0$ and $u(x) : [0, \infty) \to \mathbf{R}$ such that

$$\begin{cases} u''(x) = f(x, u(x), u'(x)) & \text{for } x \in [0, s], \\ u(0) = \alpha, \quad u'(s) = 0, & \\ u(x) = 0, & \text{for } x \in [s, \infty), \end{cases} \tag{1.1}$$

where $\alpha > 0$ and $f(x, z, p) : [0, \infty) \times \mathbf{R}^2 \to \mathbf{R}$ are given. For an application we refer to [**1**] and [**2**]. Thompson has given the following result in 1982.

Theorem 1.1. *Let α satisfy the inequality $\alpha > 0$ and suppose that $f(x, z, p)$: $[0, \infty) \times \mathbf{R}^2 \to \mathbf{R}$ satisfies the following conditions*:

1. $f(x, 0, 0) > 0$;

2. $f(x, z, p)$ *is increasing with respect to z, i.e., the inequality $z \le \tilde{z}$ implies $f(x, z, p) \le f(x, \tilde{z}, p)$;*

3. $f(x, z, p)$ *satisfies a Lipschitz condition in p on bounded subsets of its domain in $\mathbf{R}^3$.*

Then solutions to problem (1.1) *are unique when they exist.*

For the proof we refer to Corollary 1 in [**3**]. In this paper we will prove a theorem that guarantees the existence of a solution of (1.1).

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