ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 34, Number 1, Spring 2004

GENERIC FORMAL FIBERS OF POLYNOMIAL RING EXTENSIONS

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ABSTRACT. In this paper we explore the relationship between the dimension of the generic formal fiber of a Noetherian local domain R and the dimension of the generic formal fiber of the domain $R[X]$ localized at (M_R, X) where M_R is the maximal ideal of R and X is an indeterminate. Specifically, we show that if R is a universally catenary local Noetherian domain such that the dimension of the generic formal fiber of $R[X]_{(M_R,X)}$ is $\dim R$, then the dimension of the generic formal fiber of R is $\dim R - 1$. We also provide counter-examples showing that the converse does not hold.

1. Introduction. Let (R, M_R) be a local Noetherian domain with maximal ideal M_R, quotient field K and M_R-adic completion $\hat{R}$. The generic formal fiber ring of R is defined to be $\hat{R} \otimes_R K$. The dimension of the generic formal fiber of R is the Krull dimension of the generic formal fiber ring of R. In this setting we will denote the dimension of the generic formal fiber of R by $\alpha(R)$. Suppose that $\hat{P}$ is a prime ideal of $\hat{R}$ satisfying $\hat{P} \cap R = (0)$. Then we say that $\hat{P}$ is in the generic formal fiber of R.

If (R, M_R) is a complete local domain of dimension $n \geq 1$ which contains a field, Matsumura shows in [**4**, Example 1] that the dimension of the generic formal fiber of the localized polynomial ring $R[X]_{(M_R,X)}$ is $n-1$. This implies that for every local Noetherian domain (A, M_A) of dimension n the dimension of the generic formal fiber ring of $A[X]_{(M_A,X)}$ is at least $n-1$. This fact seems to indicate that there is little or no relationship between $\alpha(A)$ and $\alpha(A[X]_{(M_A,X)})$ except possibly in the case where $\alpha(A) = n-1 = \dim(A) - 1$.

Heinzer, Rotthaus and Sally informally posed the following conjecture.

Research of the first author supported by NSF grant DMS 9973069.
Research of the second author supported by NSF grant DMS 980122.
Received by the editors on May 18, 2001, and in revised form on November 26, 2001.