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A SURVEY ON RECENT ADVANCES ON THE NIKODÝM BOUNDEDNESS THEOREM AND SPACES OF SIMPLE FUNCTIONS

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ABSTRACT. In this paper we review the research about the barrelledness properties of the normed space of simple functions associated to a Boolean ring provided with the supremum-norm. We also exhibit some results concerning the barrelledness of certain closely related normed spaces of vector-valued functions. We have included an explanation of the strategy of some proofs and given account of the relevant techniques.

1. Introduction. This paper aims to survey the literature concerning normed spaces of simple functions associated to certain Boolean rings and study the barrelledness of some spaces of vector-valued bounded functions. Let us start by recalling some definitions that will be used throughout this paper. A subset A of a topological space X is said to be of *first category* if it is the union of a sequence of nowhere dense subsets of X, otherwise A is called of *second category*. A topological space is said to be *Baire* if each nonempty open subset is of second category. A subset A of a topological vector space, tvs for short, E over the field $\mathbf{K}$ of the real or complex numbers is *absolutely convex* if $\lambda x + \mu y \in A$ for each $x, y \in A$ and $\lambda, \mu \in \mathbf{K}$ such that $|\lambda| + |\mu| \leq 1$ and *absorbing* if for each $x \in E$ there exists $\lambda > 0$ such that $\lambda x \in A$. Each neighborhood of the origin of a tvs is absorbing. A tvs is called *locally convex*, lc, if there exists a base of neighborhoods of the origin formed by (closed) absolutely convex sets. Hereafter every tvs will be assumed to be Hausdorff. It can be shown that a tvs is metrizable if and only if it has a countable base of neighborhoods of the origin. A complete metrizable tvs E is called an (F)-space. If in addition E is lc, then E

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