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ARCHIMEDEAN CLOSED LATTICE-ORDERED GROUPS

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ABSTRACT. We show that, if an abelian lattice-ordered group is archimedean closed, then each principal l-ideal is also archimedean closed. This has given a positive answer to the question raised in 1965 and hence proved that the class of abelian archimedean closed lattice-ordered groups is a radical class. We also provide some conditions for lattice-ordered group $F(\Delta, R)$ to be the unique archimedean closure of $\sum(\Delta, R)$.

Introduction. Throughout, let G be a lattice-ordered group (l-group).

Let Γ be a *root system*, that is, Γ is a partially ordered set for which $\{\alpha \in \Gamma \mid \alpha \geq \gamma\}$ is totally ordered, for any $\gamma \in \Gamma$. Let $\{H_\gamma \mid \gamma \in \Gamma\}$ be a collection of abelian totally-ordered groups indexed by Γ. $V(\Gamma, H_\gamma)$ is the set of all functions v on Γ for which $v(\gamma) \in H_\gamma$ and the support of each v satisfies ascending chain condition. $V(\Gamma, H_\gamma)$ is an abelian group under addition. Furthermore, if we define an element of $V(\Gamma, H_\gamma)$ to be positive, if it is positive at each maximal element of its support, then $V(\Gamma, H_\gamma)$ is an abelian l-group, which we call a *Hahn group* on Γ. $\sum(\Gamma, H_\gamma)$ is the l-subgroup of $V(\Gamma, H_\gamma)$ whose elements have finite supports. A *root* in a root system Γ is a totally ordered subset of Γ. $F(\Gamma, H_\gamma)$ is the l-subgroup of $V(\Gamma, H_\gamma)$ such that the support of each element is contained in a finite number of roots in Γ.

A convex l-subgroup which is maximal with respect to not containing some $g \in G$ is called *regular* and is a *value* of g. Element g is *special* if it has a unique value, and in this case the value is called a *special value*. A convex l-subgroup P of G is *prime* if $a \wedge b = 0$ in G implies that either $a \in P$ or $b \in P$. Regular subgroups of G are prime and form a root system under inclusion, written $\Gamma(G)$. A subset $\Delta \subseteq \Gamma(G)$ is *plenary* if $\cap\Delta = \{0\}$ and Δ is a dual ideal in $\Gamma(G)$; that is, if $\delta \in \Delta$, $\gamma \in \Gamma(G)$ and $\gamma > \delta$, then $\gamma \in \Delta$. If G is an abelian l-group, then G is l-isomorphic to

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