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OSCILLATIONS OF SECOND-ORDER NONLINEAR PARTIAL DIFFERENCE EQUATIONS

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ABSTRACT. Oscillations of the second-order nonlinear partial difference equation

$$T(\triangle_1, \triangle_2)\,[c_{mn}T(\triangle_1, \triangle_2)(y_{mn})] + p_{mn}(y_{m+1,n} + y_{m,n+1})^{\nu} = 0$$

is investigated. Some sufficient conditions for oscillations of solutions of the above equation with $\nu > 1$ and $\nu < 1$ are obtained, where ν is a fraction of odd positive integers, $m, n \in N_i = \{i, i+1, \dots, \}$, i is a nonnegative integer, $T(\triangle_1, \triangle_2) = \triangle_1 + \triangle_2 + I$, $\triangle_1 y_{mn} = y_{m+1,n} - y_{mn}$, $\triangle_2 y_{mn} = y_{m,n+1} - y_{mn}$, $I_{mn} y_{mn} = y_{mn}$.

1. Introduction. Partial difference equations are popular and important in many applications such as those involving population dynamics with spatial migrations, chemical reactions, etc., and also in computation and analysis of finite difference equations [**2, 3, 9, 10**]. In the past several years, the qualitative theory of partial difference equations have been extensively investigated, see [**1, 5–8, 11–17**] and references therein. In particular, oscillations of all solutions of the second order nonlinear partial difference equation

$$T(\triangle_1, \triangle_2)[c_{mn}\,\triangle_1\,(y_{mn})] + \sum_{i=1}^{s} a_i(m,n) f_i(y_{m+1,n}, \triangle_1(y_{mn})) = 0$$

have been studied [**4**], where $T(\triangle_1, \triangle_2) = \triangle_1 + \triangle_2 + I$, $\triangle_1 y_{mn} = y_{m+1,n} - y_{mn}$, $\triangle_2 y_{mn} = y_{m,n+1} - y_{mn}$ and $I(y_{mn}) = y_{mn}$. Let $N_i = \{i, i+1, \dots, \}$, where i is a nonnegative integer, $\{a_i(m,n)\}_{(m,n)\in N_0^2}$ are real double sequences, $i = 1, 2, \dots, s$, and s is a positive integer, the double sequence $\{c_{mn}\}_{(m,n)\in N_0^2}$ is assumed to be positive.

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