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TRANSVERSALITY THEOREMS FOR HARMONIC FORMS

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ABSTRACT. We prove that harmonic 1-forms and 2-forms for generic metrics on a closed manifold M enjoy the standard transversality properties with respect to the stratification of $\wedge^k T^*M$ which is induced from the action of $O(n)$ on $\wedge^k(\mathbf{R}^n)^*$.

1. Introduction. This paper is motivated by the following (related) questions on the regularity of the zero sets of solutions to Laplace's equation:

(A) *The Dirichlet problem.* Let Ω be a domain in $\mathbf{R}^n$ and Δ_g be the Laplacian on Ω with respect to the (Riemannian) metric g. Consider the solution u to the equation $\Delta_g u = 0$, with Dirichlet boundary condition $u|_{\partial\Omega} = f$, where f is a fixed function on $\partial\Omega$. Then is the zero set of u regular for generic choices of g?

(B) *Harmonic forms.* Let M be a closed, oriented n-manifold and Δ_g be the Laplacian (Laplace-Beltrami operator) on M with respect to the (Riemannian) metric g. If g is a generic metric and ω is a harmonic k-form with respect to the metric g (i.e., $\Delta_g\omega = 0$), is the zero set of ω regular? Moreover, is the generic harmonic form ω transverse to the various strata of $\bigwedge^k T^*M$ under the action of $SO(n)$?[1]

We will call a harmonic form ω with respect to the metric g a *g-harmonic form.* The goal of this paper is to prove affirmative results for (A) and certain special cases of (B). On a closed, oriented n-manifold M we prove transversality results for 1-forms and also for 2-forms (provided n is even). Dually, we obtain transversality for $(n-1)$-forms on any n-manifold and $(n-2)$-forms for n even. As we shall see, 4-manifolds exhibit unusual behavior in the dichotomy between the self-dual (SD) (or anti-self-dual (ASD)) 2-forms and the non-SD (and non-ASD) 2-forms. The following is a sampling of the results which are proved:

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